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PRESSURE FILTRATION IN A CRACKED AND POROUS STRATUM

A. M. Ametov

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The main assumptions of the theory for nonsteady-state filtration in a cracked and porous material are given in [1]. A general solution is given in the present work for the first and second boundary problems of filtration in cracks.

1. We assume that pressure equals zero in a cracked and porous material occupying a half-space $x \geq 0$. From instant $t = 0$ at boundary $x = 0$ pressure starts to change by the rule $p_1(t, 0) = f(t)$. The distribution of pressure in cracks is determined from solution of the problem in [1]

$$\frac{\partial p_1}{\partial t} = \varkappa \frac{\partial^2 p_1}{\partial x^2} + \eta \frac{\partial^3 p_1}{\partial x^2 \partial t}; \tag{1.1}$$

$$p_1(t,0) = f(t); \tag{1.2}$$

$$\varkappa \frac{\partial^2 p_1(0,x)}{\partial x^2} - Ap_1(0,x) = -Ap_2(0,x);$$
$$p_2(0,x) = 0,\ p_1(0,0) = f(0). \tag{1.3}$$

Here p_1, p_2 are the pressures in cracks and blocks; $\varkappa$, η, A are coefficients introduced in [1] where it was shown that the initial pressure distribution in cracks should be found from problem (1.3) whose solution is

$$p_1(0,x) = f(0)\exp(-x/\sqrt{\eta}). \tag{1.4}$$

It is easy to prove; the solution of the first boundary problem (1.1), (1.2), (1.4) is a function

$$p_1(t,x) = \frac{2\varkappa}{\pi} \int_0^t f(u) \int_0^\infty \exp(-\varkappa(t-u)\beta^2/(1+\eta\beta^2)) \frac{\beta \sin(x\beta)}{(1+\eta\beta^2)^2}\, d\beta du + f(t)\exp(-x/\sqrt{\eta}). \tag{1.5}$$

With $\eta \to 0$ problem (1.1), (1.2), (1.4) is converted into the first boundary problem for the piezoelectric conductivity equation [2, Eq. (861.21)]

$$\frac{\partial p_1}{\partial t} = \varkappa \frac{\partial^2 p_1}{\partial x^2},\ p_1(t,0) = f(t),\ p_1(0,x) = 0, \tag{1.6}$$

and its solution is converted into the solution of problem (1.6)

$$\lim_{\eta \to 0} \left[\frac{2\varkappa}{\pi} \int_0^t f(u) \int_0^\infty \exp(-\varkappa(t-u)\beta^2/(1+\eta\beta^2)) \frac{\beta \sin(x\beta)}{(1+\eta\beta^2)^2}\, d\beta du + f(t)\exp(-x/\sqrt{\eta}) \right] = \frac{2\varkappa}{\pi} \int_0^t f(u) \int_0^\infty \exp(-\varkappa(t-u)\beta^2)\beta \sin(x\beta)\, d\beta du =$$
$$\frac{x}{2\sqrt{\pi\varkappa}} \int_0^t f(u)(t-u)^{-3/2} \exp(-x^2/4\varkappa(t-u))\, du.$$

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It is noted that the pressure in blocks is determined from the equation in [1]

$$p_2(t,x) = p_1(t,x) - \frac{\varkappa}{A}\frac{\partial^2 p_1(t,x)}{\partial x^2}.$$

2. We consider the problem of flow towards a drainage gallery. Then conditions (1.2), (1.4), and solution (1.5) are written in the form

$$p_1(t,0) = p_0 = \text{const},\ p_1(0,x) = p_0\exp(-x/\sqrt{\eta}),$$
$$p_1(t,x) = p_0\Big[1 - \frac{2}{\pi}\int_0^\infty \exp\Big(-\frac{\varkappa t}{\eta}\frac{\beta^2}{1+\beta^2}\Big)\frac{\sin((x/\sqrt{\eta})\beta)}{\beta(1+\beta^2)}d\beta\Big]. \quad (2.1)$$

We calculate the flow of liquid through boundary x = 0. By differentiating expression (2.1) with respect to x with x = 0 we obtain

$$q = -\frac{k_1}{\mu}\frac{\partial p_1}{\partial x}\Big|_{x=0} = \frac{2p_0k_1}{\pi\sqrt{\eta}\mu}\int_0^\infty \exp\Big(-\frac{\varkappa t}{\eta}\frac{\beta^2}{1+\beta^2}\Big)\frac{d\beta}{1+\beta^2}, \quad (2.2)$$

where k_1 is crack permeability; μ is liquid viscosity. By using the Laplace method (see for example [3]) we find asymptotic expressions for pressure (2.1) with small x and flow (2.2) with $t \to \infty$:

$$p_1(t,x) \sim p_0\Big(1 - \frac{x}{\sqrt{\pi\varkappa t}}\Big(1 + \frac{\eta}{4\varkappa t}\Big)\Big),\ q \sim \frac{p_0}{\sqrt{\pi\varkappa t}}\Big(1 + \frac{\eta}{4\varkappa t}\Big). \quad (2.3)$$

It can be seen from expression (2.3) that with filtration in porous material ($\eta = 0$) the pressure will be greater and the flow will be less than with a cracked and porous material. This is connected with the fact that as a result of exchange of liquid between blocks and cracks liquid entering a boundary is partly released by blocks adjacent to it. It is found that blocks are 'run-offs' for pressure from cracks and 'sources' of liquid for cracks. With $t < \eta/\varkappa$ from relationship (2.2) we obtain

$$q(t) \sim \frac{p_0k_1}{\sqrt{\eta}\mu}\Big(1 - \frac{1}{2}\frac{\varkappa t}{\eta}\Big). \quad (2.4)$$

3. We find pressure distribution with prescribed flow of liquid through a boundary (second boundary problem). For this we replace boundary čondition (1.2) as flows:

$$\frac{\partial p_1(t,0)}{\partial x} = -\frac{\mu}{k_1}q(t). \quad (3.1)$$

Correspondingly initial condition (1.4) is also changed:

$$p_1(0,x) = \frac{\mu}{k_1}q(0)\sqrt{\eta}\exp(-x/\sqrt{\eta}). \quad (3.2)$$

It is easy to prove that the solution of the problem (1.1), (3.1), (3.2) is given by the equation

$$p_1(t,x) = \frac{2\varkappa\mu}{\pi k_1}\int_0^t q(u)\int_0^\infty \exp\Big(-\varkappa(t-u)\frac{\beta^2}{1+\eta\beta^2}\Big)\frac{\cos(x\beta)}{(1+\eta\beta^2)^2}d\beta du + \frac{\mu}{k_1}q(t)\sqrt{\eta}\exp(-x/\sqrt{\eta}). \quad (3.3)$$

with $q(t) = q_0$ = const from relationship (3.3) we have

$$p_1(t,x) = \frac{\mu}{k_1}q_0\Big[\frac{2}{\pi}\int_0^\infty \frac{1-\exp(-\varkappa t\beta^2/(1+\eta\beta^2))}{\beta^2}\frac{\cos(x\beta)}{1+\eta\beta^2}d\beta + \sqrt{\eta}\exp(-x/\sqrt{\eta})\Big]. \quad (3.4)$$

With $t < \eta/\varkappa$ it follows from relationship (3.4) that

$$p_1(t,0) \sim \frac{\mu}{k_1}q_0\sqrt{\eta}\Big(1 + \frac{1}{2}\frac{\varkappa t}{\eta}\Big). \quad (3.5)$$

It can be seen by comparing Eqs. (2.4) and (3.5) that they are similar to each other since the latter may be converted

$$q_0 \sim \frac{p_1(t,0)k_1}{\sqrt{\eta}\mu} \frac{1}{1 + \frac{1}{2}\frac{\varkappa t}{\eta}} \sim \frac{p_1(t,0)k_1}{\sqrt{\eta}\mu}\left(1 - \frac{1}{2}\frac{\varkappa t}{\eta}\right),$$

and from them it follows that in order to maintain a constant flow of liquid through the boundary pressure should increase linearly, but with a fixed pressure in the gallery flow decreases linearly.

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